Anyone who intends to use the PLTMG software will need a copy of this book. Despite the aforementioned deficiencies, the book contains all the information needed to solve complicated boundary value problems, and is written in a clear, easy-to-understand style.

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23[65M55, 65N55].—P. W. HEMKER & P. WESSELING (Editors), *Contributions to Multigrid*, CWI Tract, Vol. 103, Centre for Mathematics and Computer Science, Amsterdam, 1994, viii + 220 pp., 24 cm. Price: Softcover Dfl. 60.00.

From the Preface: This volume contains a selection from the papers presented at the Fourth European Multigrid Conference, held in Amsterdam, July 6–9, 1993.

J. H. B.

24[65L05].—LAWRENCE F. SHAMPINE, *Numerical Solution of Ordinary Differential Equations*, Chapman & Hall, New York, 1994, x + 484 pp., $23\frac{1}{2}$ cm. Price $64.95.

That so many books with more or less this same title have appeared in recent years might lead one to expect nothing new in this volume. This is far from the case. Numerical methods for differential equations is a very difficult and important subject and, while its literature is extremely rich, it is far from mature. Recent textbooks, and more so monographs, are not so much personal expositions of a well-defined body of work but personal contributions to the development of a vital and rapidly-changing research area. Lawrence Shampine has been a significant contributor to the theory and practice of solving differential equations numerically for 20 years and it is his style, developed and honed through his own research and experience, that is stamped on this book.

The book is divided into eight chapters of which the first three are of an introductory nature. The first deals with "The Mathematical Problem" (of solving ordinary differential equations), the second with "Discrete Variable Methods", and the third "The Computational Problem". Chapter four on "Basic Methods" is followed by the theory of "Convergence and Stability". The last three chapters are on "Stability for Large Step Sizes", "Error Estimation and Control" and "Stiff Problems".

References are collected together for standard literature, works actually cited, and codes referred to. The book concludes with a brief appendix on mathematical tools used in the book.

The subject of solving differential equations numerically is a mix of theoretical knowledge, practical insight and computational technique. In the Shampine style, software is also an essential component and it is the balance in emphasis between this and the other aspects of the subject that makes this book especially attractive.

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25[34C35, 58F05, 65L06, 65L07].—J. M. SANZ-SERNA & M. P. CALVO, *Numerical Hamiltonian Problems*, Applied Mathematics and Mathematical Computation, Vol. 7, Chapman & Hall, London, 1994, xii + 207 pp., 22 cm. Price \$48.50.

The story of numerical analysis is mostly about translating mathematical concepts and models into a quantitative medium, fleshing numbers on the formal mathematical skeleton. This, however, should not obscure the crucial role of *qualitative* aspects of computation. It is an illusion that, as soon as it comes to application, all questions are of a purely quantitative character. 'Will the satellite stay in stable orbit?', 'Does a mixture undergo combustion?', 'Will the species survive in a given environment?', are all qualitative questions. Presumably, they are modelled by differential equations. Presumably, these differential equations are solved numerically. Certainly, their numerical solution contains errors. Naively, the purpose of numerical analysis is to minimize the accretion of error, but this frequently misses the point of the whole calculation. Thus, let us consider the stability of a satellite and suppose that two alternative computational methods are available. The first produces a very small error which, however, consistently undershoots the elevation. The second is considerably more error-prone but gets the stability issue exactly right: the numerical orbit is stable if and only if so is the exact one. Little doubt that, for the specific purpose in hand, the second method is superior!

The emphasis on the recovery of qualitative attributes of a mathematical model, rather than just minimizing the error, is relatively a new one. It has led in the last decade to a profound new insight into computation and has changed the treatment of many important numerical problems. Arguably, the most significant advance has been associated with Hamiltonian problems.

A system of ordinary differential equations is said to be Hamiltonian if it can be represented in the form

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H(\mathbf{p}, \mathbf{q})}{\partial \mathbf{q}},$$
$$\frac{d\mathbf{q}}{dt} = \frac{\partial H(\mathbf{p}, \mathbf{q})}{\partial \mathbf{p}},$$

where H is a given C^1 function. A significant proportion of dynamical systems that occur in mechanics—classical and quantum alike—can be rendered in a Hamiltonian form. As Penrose comments, "Such unity of form in the structure of dynamical equations, despite all the revolutionary changes that have occurred in physical theories over the past century or so is truly remarkable!" [3].

The formulation of equations of motion by William Rowan Hamilton in the above form was highly significant for sound physical reasons. The letter $\mathbf{p}$ stands for positions and $\mathbf{q}$ for momenta of physical particles, whilst H is